Necessity, Possibility, Equivalence, and Consequence (NPEC)

Phil 102: Logic | Handout 5
Mark Criley | 09/17/2018

NPEC: The General Ideas

- A sentence P is necessary if and only if P must be true (i.e., could not be false).
- A sentence P is possible if and only if P could be true (i.e., is not forced to be false)
- Sentences P and Q are equivalent if and only if they must always have the same truth value.
- A sentence Q is a consequence of a set of other sentences P_1, P_2, … if and only if Q must be true (could not be false) whenever all of P_1, P_2, … are true.

Varieties of Necessity, Possibility, Equivalence, and Consequence

The above are fine, intuitive general formulations, but we would like to have a better sense of what it means to say that a sentence *must* be true, or is *forced* to be false. What does the forcing? As it turns out, there are a number of different "background facts" that might play that role.

The meanings of the Boolean connectives impose some constraints the truth or falsity of sentences. For instance, if a sentence is true, its negation must be false. The *general meanings of the predicates of FOL impose some more constraints.* For instance, if a=b, then it must be that b=a, and it must be that a and b are the same size. But a truth table couldn't tell you that. Boole doesn't know anything about what "=" means, or what "SameSize" means. *The specific rules and limitations of the Tarski's World program* impose even more constraints. For instance, if "Smaller(a,b)" and "Medium(b)" are true in Tarski's World, then it must be that "Small(a)" is true in Tarski's World. But truth tables and the general meanings of logic alone wouldn't tell you that—you'd have to know that in Tarski's World there are only the three sizes available.

Each one of these sets of "background facts" generates a corresponding type of necessity, etc.

1. Truth-Table, or TT-, or Tautological NPEC

- P is TT-necessary (a tautology) if and only if <u>given the meanings of the Boolean connectives</u>, P must be true (i.e., could not be false)—that is, if and only if P is true on every row of its truth table.
- P is TT-possible if and only if <u>given the meanings of the Boolean connectives</u>, P could be true (i.e., is not forced to be false)—that is, if and only if P is true in at least one row of its truth table.

- P and Q are TT-equivalent if and only if given the meanings of the Boolean connectives, they must have the same truth value—that is, if and only if they have exactly the same columns of truth values under their main connectives in their joint truth table.
- Q is a TT-consequence of a set of sentences P_1, P_2, … if and only if given the meanings of the Boolean connectives, Q must be true (could not be false) whenever P_1, P_2, … are all true—that is, if and only if there is no "counterexample" row of their joint truth table where P_1, P_2, … are all true and Q is false.

2. Logical NPEC

- P is logically necessary if and only if given the meanings of its Boolean connectives and the generally understood meanings of its predicates, P must be true (cannot be false).
- P is logically possible if and only if given the meanings of its Boolean connectives and the generally understood meanings of its predicates, P could be true (i.e., is not forced to be false).
- P and Q are logically equivalent if and only if given the meanings of their Boolean connectives and the generally understood meanings of their predicates, P and Q must have the same truth value.
- Q is a logical consequence of a set of sentences P_1, P_2, … if and only if given the meanings of their Boolean connectives and the generally understood meanings of their predicates, Q must be true (could not be false) whenever P_1, P_2, … are all true.

3. Tarski's World (TW-) NPEC

- P is TW-necessary if and only if given the meaning of the Boolean connectives, the meanings of the predicates, and the rules and limitations of Tarski's World, P must be true (i.e., could not be false)—that is, if and only if P is true in every Tarksi's World world that could be built.
- P is TW-possible if and only if given the meaning of the Boolean connectives, the meanings of the predicates, and the rules and limitations of Tarski's World, P could be true (i.e., is not forced to be false)—that is, if and only if P is true in at least one Tarski's World world.
- P and Q are TW-equivalent if and only if given the meaning of the Boolean connectives, the meanings of the predicates, and the rules and limitations of Tarski's World, P and Q must have the same truth value—that is, if and only if P and Q have the same truth value in every world that could be built in Tarski's World.
- Q is a TW-consequence of a set of sentences P_1, P_2, … if and only if given the meaning of the Boolean connectives, the meanings of the predicates, and the rules and limitations of Tarski's World, Q must be true (could not be false) whenever P_1, P_2, … are all true—that is, if and only if there is no Tarski's World world in which P_1, P_2, … are all true and Q is false.

There are other categories of "background fact" that we won't be studying, which would generate different sorts of necessity, possibility, consequences, and equivalence. (Example: What constraints are imposed by the laws of physics? P is physically necessary if and only if given the laws of physics it must be true.)

Consequence and Validity

Remember that the notion of *consequence* is intimately connected with the notion of *the validity of an argument*. An argument is valid if and only if its conclusion is a consequence of its premises. An argument is invalid if and only if its conclusion is not a consequence of its premises. Thus, we can ask of an argument whether it is TT-valid, logically valid, or TW-valid, and the answer will correspond to the type of consequence we're asking about. (Exam #1 has a problem that asks you to use a truth table to assess the truth table validity of an argument.)