

## The Proof Rules

### Intro Rules

#### $\wedge$ -Intro

$$\left| \begin{array}{l} P_1 \\ \vdots \\ P_n \end{array} \right. \\ P_1 \wedge \dots \wedge P_n$$

#### $\vee$ -Intro

$$\left| \begin{array}{l} P_i \\ \vdots \\ P_1 \vee \dots \vee P_i \dots \vee P_n \end{array} \right.$$

#### $\neg$ -Intro

$$\left| \begin{array}{l} \left| \begin{array}{l} P \\ \vdots \\ \perp \end{array} \right. \\ \neg P \end{array} \right.$$

#### $\perp$ -Intro

$$\left| \begin{array}{l} P \\ \vdots \\ \neg P \\ \perp \end{array} \right.$$

### Elim Rules

#### $\wedge$ -Elim

$$\left| \begin{array}{l} P_1 \dots \wedge P_i \wedge \dots P_n \\ \vdots \\ P_i \end{array} \right.$$

#### $\vee$ -Elim

$$\left| \begin{array}{l} P_1 \vee \dots \vee P_n \\ \left| \begin{array}{l} P_1 \\ \vdots \\ R \end{array} \right. \\ \vdots \\ \left| \begin{array}{l} P_n \\ \vdots \\ R \end{array} \right. \\ R \end{array} \right.$$

#### $\neg$ -Elim

$$\left| \begin{array}{l} \neg \neg P \\ \vdots \\ P \end{array} \right.$$

### $\perp$ -Elim

$$\left| \begin{array}{l} \perp \\ \vdots \\ P \end{array} \right.$$

### Other Rules

#### Reiteration

$$\left| \begin{array}{l} P \\ \vdots \\ P \end{array} \right.$$

#### Analytic Consequence “Ana Con”

Ana Con is a very powerful, “wild-card” rule in FITCH. It allows you to select any earlier steps in your proof and introduce any logical consequence of them. Because it is so powerful, it is usually restricted so that you can only to “literals”—a literal is an atomic sentence or the negation of an atomic sentence.

See the other side of this sheet for helpful strategies, guidelines and hints for constructing proofs!

## Strategies, Guidelines, and Hints for Proofs

1. Before you start to use FITCH, think about what the premises and conclusion of the argument mean. Try to step through the reasoning informally, talking yourself through it, asking questions about what follows from what. (“Okay, so I know that  $a$  is either a cube or a dodec ...”)
2. When you use FITCH, start by trying to work from the bottom up. See what your conclusion is. What’s its main connective? What other steps would you need to have earlier in the proof in order to use the corresponding intro rule in order to get that connective? You might even try just adding those steps to the proof right above the conclusion, and then using that intro rule to get the conclusion. This won’t *always* work. But it often does. And when it does, you have made your remaining problem simpler: now all you have to do is try to figure out how to introduce those steps you just added.
3. When you’ve done as much as you can from the bottom up, try working from the top down. Look at your premises. What are their main connectives? Are you in a position to use any of the corresponding elim rules on those premises? Repeat this process for the various sentences you have to work down from. If you have a premise that you haven’t used in your proof so far, that’s a clue that you should try to use it in an elim rule.
4. Never start a subproof without knowing which rule you are planning to use it for. There are only two rules we know so far that use subproofs:  $\neg$ -Intro and  $\vee$ -Elim. You don’t need to use a subproof for any other rule.
5. Once you decide to use one of the rules that requires a subproof, always fill in the “skeleton” of the subproof:
  - **You know how many subproofs you are going to need.** For  $\neg$ -Intro, you only need one. For  $\vee$ -Elim, you have to have one for each disjunct.
  - **You know what goes on the “assumption shelf” for each of these subproofs.** For  $\neg$ -Intro, when you’re trying to prove  $\neg P$ , you assume  $P$ . For  $\vee$ -Elim, you assume each of the disjuncts of the disjunction in its own subproof.
  - You know that you are going to need to add in some spaces after that assumption shelf to give yourself room to work.
  - **You have a good idea of what you should put on the last line of the subproof.** For  $\neg$ -Intro, you know that last line is always going to be  $\perp$ . For  $\vee$ -Elim, it is usually a good idea to write the conclusion of the whole argument. (Why not go for the whole enchilada? If it works, you’ll be done with the whole proof!) Remember that for  $\vee$ -Elim, you have to have **exactly** the same sentence at the end of each subproof.
  - **And you know what goes on the first step outside of the subproof.** For  $\neg$ -Intro, you know it’s going to be the sentence that you assumed with a negation in front of it. For  $\vee$ -Elim, it has to be the line that you’ve repeated at the end of each subproof.

Filling in the “skeleton” of the proof makes a **huge difference**. It’s just too easy to lose track of what you’re doing in a proof because you think to yourself, “I’ll just fill in that step later.” Fill it in now, while you’re thinking of it. This provides you with more to work with and will trigger new insights. And since partial credit is based on the percentage of a correct proof you have in your file, it’s crazy not to fill in this information right away and lock down those points.

See the other side for a summary of the proof rules.