The Proof Rules (Updated)

Intro Rules

$\wedge$ Intro

P_1
$\vdots$
P_n
$P_1 \wedge \ldots \wedge P_n$

$\vee$ Intro

P_i
$\vdots$
$P_1 \vee \ldots \vee P_i \ldots \vee P_n$

$\neg$ Intro

| P
| $\vdots$
| $\bot$
$\neg P$

$\bot$ Intro

P
$\vdots$
$\neg P$
$\bot$

$\rightarrow$ Intro

| P
| $\vdots$
| Q
$P \rightarrow Q$

$\leftrightarrow$ Intro

| P
| $\vdots$
| Q
| Q
| $\vdots$
| P
$P \leftrightarrow Q$

Elim Rules

$\wedge$ Elim

$P_1 \ldots \wedge P_i \wedge \ldots P_n$
$\vdots$
P_i

$\vee$ Elim

$P_1 \vee \ldots \vee P_n$
| P_1
| $\vdots$
| R
$\vdots$
| P_n
| $\vdots$
| R
R

$\neg$ Elim

$\neg\neg P$
$\vdots$
P

$\bot$ Elim

$\bot$
$\vdots$
P

$\rightarrow$ Elim

$P \rightarrow Q$
$\vdots$
P
$\vdots$
Q

$\leftrightarrow$ Elim

$P \leftrightarrow Q$ (…or $Q \leftrightarrow P$)
$\vdots$
P
$\vdots$
Q

Other Rules

Reiteration

P
$\vdots$
P

Analytic Consequence
"Ana Con"

This rule allows you to introduce any logical consequence of earlier steps. Because it is so powerful, Ana Con is usually restricted so that you can only to "literals"—a literal is an atomic sentence or the negation of an atomic sentence.