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Here are a couple more FO Invalid arguments for you to use as practice. (These are not an assignment; they are never due.)

Both of the following arguments are FO Invalid. Give a First Order Counterexample for each.

Argument 1

$\forall x\ (\mathsf{Cube}(x) \to \mathsf{Small}(x))$
$\forall x\ (\mathsf{Small}(x) \to \mathsf{Adjoins}(b,x))$
$\forall x\ (\mathsf{Cube}(x) \to \mathsf{Adjoins}(x,b))$

Argument 2

$\exists x\ \mathsf{Dodec}(x) \to \forall x\ \mathsf{Medium}(x)$
$\neg\forall x\ \neg\mathsf{Dodec}(x)$
$\neg\exists x\ \mathsf{Small}(x)$

In order to show that an argument is First Order Invalid, it isn't enough to put in nonsense predicates. Nonsense predicates are okay for *testing* whether an argument is FO-invalid. But in order to *demonstrate* that an argument is FO-invalid, you have to do a counterexample, like we did with the Silver Chandelier example in class. That is, you have to swap in *meaningful* predicates and names instead of nonsense ones. And you must choose predicates and names that make the premises all true and the conclusion false. Then rewrite the argument using these sensible predicates. **This rewritten argument, with true premises and a false conclusion, is your FO-counterexample.** If it is not completely obvious why your choice of predicates and names makes the premises true and the conclusion false, write a brief explanation or story that tells the reader why they fit this pattern.

Want some more? In class, I am revealing three of the arguments in the textbook that are FO-Invalid. Write them below as I list them, and then you can practice doing FO Counterexamples with them:

I'd be happy to work with you on them in office hours, or to check your work on them after you're done.

Practice For **Exercise 10.9**

Here are four sentences we will use as practice for Exercise 10.9:

- $\forall x\,(\mathsf{Cube}(x) \to \neg\mathsf{Tet}(x))$
- $\exists y\,\mathsf{Smaller}(y,b) \lor \exists y\,\mathsf{Larger}(y,b)$
- $\exists x\,\mathsf{Smaller}(x,b) \to \exists x\,(x \neq b)$
- $\forall x\,\mathsf{Cube}(x) \lor \exists x\,\neg\mathsf{Cube}(x)$

Here are some instructions that parallel the instructions for Exercise 10.9 but with my preferred terminology used instead. I prefer to talk about logical necessities (rather than logical *truths*) and FO-necessities (rather than FO *validities.*)

1. Paraphrase each sentence in clear, colloquial English.
2. For each sentence, decide whether you think it is a logical ~~truth~~ necessity. If it isn't, build a world in which the sentence comes out false.
3. Which of these sentences are first-order ~~validities~~ necessities?
4. For the remaining sentence(s) (logical ~~truths~~ necessities but not first-order ~~validities~~ necessities), apply the Sensible Replacement Method to come up with a first-order counterexample (that makes the sentence false). Make sure you describe both your interpretations of the predicates and the falsifying circumstance.