

## Old Rules

### $\neg$ -Intro

P
⋮
⊥
$\neg P$

### $\rightarrow$ -Intro

P
⋮
Q
$P \rightarrow Q$

### $\leftrightarrow$ -Intro

P
⋮
Q
Q
⋮
P
$P \leftrightarrow Q$

### Reiteration

P
⋮
P

## Quantifier Rules

### Easier Rules

These two rules don't require sub-proofs or our funny boxing. Also, the particular choice of name does not matter.

### $\forall$ -Elim

$\forall x P(x)$
⋮
P(a)

### $\exists$ -Intro

P(a)
⋮
$\exists x P(x)$

### Harder Rules

These two rules require that we box up a unique name—a name that does not appear outside of the subproof—and use it as a sort of “nickname.”

### $\exists$ -Elim

$\exists x P(x)$			
<table style="border-collapse: collapse; margin-left: 20px;"> <tr><td style="border-left: 1px solid black; padding-left: 5px;"><span style="border: 1px solid black; padding: 2px;">a</span> P(a)</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">⋮</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">Q</td></tr> </table>	<span style="border: 1px solid black; padding: 2px;">a</span> P(a)	⋮	Q
<span style="border: 1px solid black; padding: 2px;">a</span> P(a)			
⋮			
Q			
Q			

### $\forall$ -Intro

<table style="border-collapse: collapse; margin-left: 20px;"> <tr><td style="border-left: 1px solid black; padding-left: 5px;"><span style="border: 1px solid black; padding: 2px;">a</span></td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">⋮</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">P(a)</td></tr> </table>	<span style="border: 1px solid black; padding: 2px;">a</span>	⋮	P(a)
<span style="border: 1px solid black; padding: 2px;">a</span>			
⋮			
P(a)			
$\forall x P(x)$			

### $\forall$ -Intro (Souped Up Version, with $\rightarrow$ -Intro Baked In!)

<table style="border-collapse: collapse; margin-left: 20px;"> <tr><td style="border-left: 1px solid black; padding-left: 5px;"><span style="border: 1px solid black; padding: 2px;">a</span> S(a)</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">⋮</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 5px;">P(a)</td></tr> </table>	<span style="border: 1px solid black; padding: 2px;">a</span> S(a)	⋮	P(a)
<span style="border: 1px solid black; padding: 2px;">a</span> S(a)			
⋮			
P(a)			
$\forall x (S(x) \rightarrow P(x))$			

### Taut Con

This wild card rule allows you to write down any Tautological (i.e., Truth Table) consequence of earlier steps. You are now allowed to use this rule on problem sets and Exam 3, and you may apply it to any number of steps of whatever type of complexity. This will save you from having to do some of the more tedious older connective rules (like  $\forall$ -Elim) that you have already been tested on. Note that the older connective rules listed on this sheet will remain useful even with Taut Con in place.