Translations Requiring Paraphrasing

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Hard Translations

There are some sentences that do not work when you try to translate them by using the “step by step” method.

Suppose you read the following sentence in a brochure from a dance studio.

Example 1: A student who studies hard will learn to tango.

Let’s translate this using the step-by step method.

A student who studies hard will learn to tango.

Step by Step Method:

1. **Type:** Partial Inclusion?
2. **Form:** $\exists x$ (Subject(x) $\wedge$ Predicate(x))
3. **Identify Subject and Predicate:**
   - SUBJECT: x is a student who studies hard
   - PREDICATE: x learns to tango
4. **Translate Subject and Predicate:**
   - SUBJECT: Student(x) $\wedge$ StudiesHard(x)
   - PREDICATE: LearnsTango(x)
5. **Substitute Translations**
   - $\exists x$ ((Student(x) $\wedge$ StudiesHard(x)) $\wedge$ LearnsTango(x))

$\exists x$ ((Student(x) $\wedge$ StudiesHard(x)) $\wedge$ LearnsTango(x))

But now, let’s check our work:

6. Read It Back!
   - There is a student who studies hard and who learns how to tango.

“A tangoing student shall appear from the West …”

There is a student who studies hard and who learns to tango.

That is a very weak claim.

It doesn’t seem to fit the spirit of a dance studio advertisement.

It sounds more like a prediction from Nostradamus or the Psychic Friends hotline.

You can dance if you want to …

The dance studio doesn’t want to tell us that *at least one* student who studies hard will learn to tango.

They’re trying to convince us that *every* student who studies hard will learn to tango!

So let’s change the quantifier from $\exists$ to $\forall$.

- $\forall x\ ((\text{Student}(x) \land \text{StudiesHard}(x)) \land \text{LearnsTango}(x))$

But now there’s a problem with this translation. (What is it?)

Everything is a studying, tangoing student …?

$$\forall x\ ((\text{Student}(x) \land \text{StudiesHard}(x)) \land \text{LearnsTango}(x))$$

The above sentence is just too strong: it says that everything is a student who studies hard and learns tango.

But that is obviously false. And it isn’t what the dance studio wanted to say.

How can we weaken it?

$\forall x\ ((\text{Student}(x) \land \text{StudiesHard}(x)) \land \text{LearnsTango}(x))$

Let’s switch the $\land$ inside to a $\rightarrow$.

$$\forall x\ ((\text{Student}(x) \land \text{StudiesHard}(x)) \rightarrow \text{LearnsTango}(x))$$

Now it reads, in English:

Every student who studies hard will learn to tango.

And that **is** what we were looking for.

Step-by-Step & Repair

What we just performed is the first way of approaching these more challenging translations. I'll call this the "Step-By-Step and Repair" method.

We translated the sentence using the step by step method. But the translation that we got didn't work.

So we had to tinker with it. We changed the quantifier to get that right, and then we had to change the interior connective.

Another Method: Paraphrasing In Advance

If you don't like the Step-By-Step and Repair approach, we could take another tack.

Once we recognized that the sentence was making a universal rather than existential claim, we could have tried to paraphrase it in English before we did the translation.

A student who studies hard will learn to tango.

$\Downarrow$

Any student who studies hard will learn to tango.

Once we have done that paraphrasing in advance, the step-by-step method will work without any need for further adjustment afterwards.

Gaining Putin's Trust

Example 2. If someone discos with him, Putin will trust that person.

Let's translate that sentence using the "Step by Step and Repair" method.

Putin on the Ritz ...

Example 2. If someone discos with him, Putin will trust that person.

This appears to be a conditional:

- Someone discos with Putin $\rightarrow$ Putin trusts that person.

Translating the Antecedent

- Someone discos with Putin → Putin trusts that person

Antecedent: Someone discos with Putin

Translate this, using

- Discos(x,y) as a predicate to express that x discos with y
- Trusts(x,y) to express that x trusts y

Antecedent: ∃x Discos(x,putin)

Translating the Consequent

- ∃x Discos(x,putin) → Putin trusts that person

Now the consequent:

Consequent: Putin trusts that person

Now, who is it again that Putin trusts?

Ah yes. That person who discos with him.

We used the variable x for that person, so let's try using it again.

- Trusts(putin,x)

Completed Translation: First Try

- Someone discos with Putin → Putin trusts that person

Now put the translations of the antecedent and the consequent back into that conditional.

- ∃x Discos(x,putin) → Trusts(putin,x)

But now we have a big problem in this sentence.

Can you see what it is?

Ooops. Scope Troubles.

The scope of the ∃ quantifier only extends to the end of the first complete wff after it.

That means it only extends to the end of "Discos(x,putin)".

- <u>∃x Discos(x,putin)</u> → Trusts(putin,x)

Let's try putting parentheses in to make the scope go all the way to the end of the sentence.

- ∃x (Discos(x,putin) → Trusts(putin,x))

- ∃x (Discos(x,putin) → Trusts(putin,x))

But now we have another problem. What is it?

∃ and →: Don't Go Together Well

- ∃x (Discos(x,putin) → Trusts(putin,x))

Using a conditional as the main connective inside of an existential makes for a very weak sentence.

An existential sentence is true whenever we can find at least one thing that makes the wff that follows it true.
That means:

- This sentence will be true if we can find just one thing that makes the antecedent false.
  - (Just one thing that *doesn't* disco with Putin makes the sentence true!)
- It would also be true if we could find just one thing that makes the consequent true.
  - (Just one thing that Putin does trust makes the sentence true!)

Hmmmm ...

So what can we do?

- ∃x (Discos(x,putin) → Trusts(putin,x))

→ and ∀: Go together well

- ∃x (Discos(x,putin) → Trusts(putin,x))

We know that universals go better with →, so maybe we could try just switching the ∃ to a ∀.

That gives us

- ∀x (Discos(x,putin) → Trusts(putin,x))

And what does that mean when we read it back in English?

Show a dictator some love

- $\forall x$ (Discos(x,putin) $\rightarrow$ Trusts(putin,x))

Now it's just a total inclusion sentence, totally including

Subject: People who disco with Putin

within

Predicate: People Putin trusts

Filling these in we get

- Anyone who discos with Putin, Putin will trust.

Or, paraphrase first

Again, if we realized that there was something a little fishy going on, we might have tried to paraphrase in English before translating into FOL.

If someone discos with Putin, Putin will trust that person.

$\downarrow$

Anyone who discos with Putin, Putin will trust.
or
Putin will trust anyone who discos with him.

Those paraphrases preserve the meaning of the original sentence, and will allow us to do the "step by step" method in the usual way.

A Sign of Trouble

Here's a clue that a sentence might need some paraphrasing:

It appears to be a conditional, but there is a pronoun that reaches across the "$\rightarrow$"

If **someone** discos with Putin, Putin will trust **that person**.

Watch out!

And in general, you will want to watch out for uses of our usual English quantifier words ("some," "any," "a," etc.,) when they are embedded in English conditionals. They behave in non-standard ways.

Look Out Behind

Suppose your Driver's Ed teacher tells you:

Example 3. If anyone is behind you, don't back up!

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Since "any" usually means "every" and uses the universal quantifier, we might be tempted to say:

Example 3. $\forall x$ BackOf(x,me) $\rightarrow$ I don't back up!

But reading the example this way will lead to driving disaster!

Donkey Time!

The following example is due to a famous discussion by the philosopher Peter Geach.

Example 4. Any farmer who owns a donkey ~~beats~~ vaccinates it.[1]

[1] We have cleaned it up a little bit for the ASPCA. No animals were harmed during the construction of this example.

Any farmer who owns a donkey vaccinates it.

What's weird about this sentence? Well, let's try translating it using the Step-by-Step and Repair method.

Step by Step

- Any farmer who owns a donkey vaccinates it.

Step by Step Method:

1. **Type:** Total Inclusion
2. **Form:** $\forall$x (Subject(x) $\rightarrow$ Predicate(x))
3. **Identify Subject and Predicate:**
   - SUBJECT: x is a farmer who owns a donkey
   - PREDICATE: x vaccinates that donkey

Step By More Steps

Next, we translate the subject and predicate

4. **Translate the Subject and Predicate:**
   - SUBJECT: x is a farmer who owns a donkey

     $\downarrow$

     SUBJECT: Farmer(x) $\wedge$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y))

   - PREDICATE: x vaccinates that donkey

     $\downarrow$

     PREDICATE: Vaccinates(x,y)

Substitute Translations

Now, we will put those translated sentences back into the original skeleton.

5. **Substitute Translated Subject and Predicate:**

   $\forall$x ((Farmer(x) $\wedge$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y))) $\rightarrow$ Vaccinates(x,y))

But now there's a big problem with this "sentence".

The problem is, it's not a sentence at all.

Why not?

Scope Trouble Again

- $\forall$x ((Farmer(x) $\wedge$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y))) $\rightarrow$ Vaccinates(x,y))

The quantifier for y only extends to the end of the first complete wff after it.

So how far does the quantifier extend?

- $\forall$x ((Farmer(x) $\wedge$ $\underline{\exists\text{y (Donkey(y)} \wedge \text{Owns(x,y))}}$) $\rightarrow$ Vaccinates(x,y))

So the variable *y* is free when it occurs in "Vaccinates (x,y)"

$\forall$x ((Farmer(x) $\wedge$ $\underline{\exists\text{y (Donkey(y)} \wedge \text{Owns(x,y))}}$) $\rightarrow$ Vaccinates(x,y))

How can we fix this scope problem?

- How about moving all of the stuff about the donkey to the end of the sentence, so that it all comes together in the predicate?
- $\forall$x (Farmer(x) $\rightarrow$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y) $\wedge$ Vaccinates(x,y)))

Not quite right

Every farmer who owns a donkey vaccinates it.

Compare:

- $\forall$x (Farmer(x) $\rightarrow$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y) $\wedge$ Vaccinates(x,y))

How does this read back?

Every farmer owns a donkey whom he vaccinates.

But we didn't **want** to say that every farmer owns a donkey. That is not part of the original sentence.

Back to the Drawing Board

The question was:

How can we fix the scope problem in this sentence?

- $\forall$x ((Farmer(x) $\wedge$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y))) $\rightarrow$ Vaccinates(x,y))

- $\forall$x ((Farmer(x) $\wedge$ $\exists$y (Donkey(y) $\wedge$ Owns(x,y))) $\rightarrow$ Vaccinates(x,y))
- Try moving the $\exists$y in front of the sentence
  - $\forall$x $\exists$y ((Farmer(x) $\wedge$ Donkey(y) $\wedge$ Owns(x,y)) $\rightarrow$ Vaccinates(x,y))

$\exists$ and $\rightarrow$ ...

But what's wrong with this?

$\forall x\ \exists y\ ((\text{Farmer}(x) \wedge (\text{Donkey}(y) \wedge \text{Owns}(x,y))) \rightarrow \text{Vaccinates}(x,y))$

- $\exists$ and $\rightarrow$ don't go well together.
- They're very weak:
  - Find one thing that isn't a donkey that the farmer owns? The sentence turns out to be true.

How can we fix it? $\exists$ and $\wedge$ ...

We do know that the $\exists$ and the $\wedge$ go together nicely. So maybe we can change the $\rightarrow$ to a $\wedge$

- $\forall x\ \exists y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y))\ \underline{\wedge}\ \text{Vaccinates}(x,y))$

$\forall x\ \exists y\ ((\text{Farmer}(x) \wedge (\text{Donkey}(y) \wedge \text{Owns}(x,y)))\ \underline{\wedge}\ \text{Vaccinates}(x,y))$

But if we read that sentence back, we can tell that it is too strong.

What does it say?

- Everything is a farmer and each farmer owns a donkey and vaccinates it.

Everything is a farmer?

No, not everything is a farmer.

Our translation is false, even when our English sentence is true.

Once again,
we have failed.

One last shot ...

How about this? We didn't like this translation

$\forall x\ \exists y\ ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

because $\exists$ and $\rightarrow$ don't go together well.

So we changed the $\rightarrow$ to a $\land$

What else could we have done?

Change the $\exists$ to a $\forall$!

That is so crazy it just might work

$\forall x\ \exists y\ ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

- now becomes

$\forall x\ \forall y\ ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

It Works! It Works!

$\forall x\ \forall y\ ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

Read it back!

- Make any two choices of objects you like ...
- ...if the first choice is a Farmer ...
- ...and the second one is a donkey
- ...and that farmer owns that donkey
- ...then the farmer vaccinates that donkey

- Any farmer who owns a donkey vaccinates it.

$\forall x\ \forall y\ ((\text{Farmer}(x) \land \text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

Also equivalent:

$\forall x\ (\text{Farmer}(x) \rightarrow \forall y\ ((\text{Donkey}(y) \land \text{Owns}(x,y)) \rightarrow \text{Vacc}(x,y)))$

And even:

$\forall y\ (\text{Donkey}(y) \rightarrow \forall x\ ((\text{Farmer}(x) \land \text{Owns}(x,y)) \rightarrow \text{Vacc}(x,y)))$