Translations Requiring Paraphrasing

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Hard Translations

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Suppose you read the following sentence in a brochure from a dance studio.

Example 1: A student who studies hard will learn to tango.

Let's translate this using the step-by step method.

A student who studies hard will learn to tango.

Step by Step Method:

1. **Type:** Partial Inclusion?

A student who studies hard will learn to tango.

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2. **Form:** $\exists$x (Subject(x) $\wedge$ Predicate(x))

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2. **Form:** $\exists$x (Subject(x) $\wedge$ Predicate(x))
3. **Identify Subject and Predicate:**

A student who studies hard will learn to tango.

Step by Step Method:

1. **Type:** Partial Inclusion?
2. **Form:** $\exists x$ (Subject(x) $\wedge$ Predicate(x))
3. **Identify Subject and Predicate:**
   - SUBJECT: x is a student who studies hard

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   - PREDICATE: x learns to tango

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5. **Substitute Translations**

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   - $\exists x$ ((Student(x) $\wedge$ StudiesHard(x)) $\wedge$ LearnsTango(x))

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But now, let's check our work:

6. Read It Back!

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$\exists x\ ((\text{Student}(x) \wedge \text{StudiesHard}(x)) \wedge \text{LearnsTango}(x))$

But now, let's check our work:

6. Read It Back!
   - There is a student who studies hard and who learns how to tango.

"A tangoing student shall appear from the West ..."

There is a student who studies hard and who learns to tango.

"A tangoing student shall appear from the West ..."

There is a student who studies hard and who learns to tango.

That is a very weak claim.

"A tangoing student shall appear from the West ..."

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It doesn't seem to fit the spirit of a dance studio advertisement.

"A tangoing student shall appear from the West ..."

There is a student who studies hard and who learns to tango.

That is a very weak claim.

It doesn't seem to fit the spirit of a dance studio advertisement.

It sounds more like a prediction from Nostradamus or the Psychic Friends hotline.

You can dance if you want to ...

The dance studio doesn't want to tell us that *at least one* student who studies hard will learn to tango.

You can dance if you want to ...

The dance studio doesn't want to tell us that *at least one* student who studies hard will learn to tango.

They're trying to convince us that *every* student who studies hard will learn to tango!

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So let's change the quantifier from $\exists$ to $\forall$.

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So let's change the quantifier from $\exists$ to $\forall$.

- $\forall x\ ((\text{Student}(x) \wedge \text{StudiesHard}(x)) \wedge \text{LearnsTango}(x))$

You can dance if you want to ...

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They're trying to convince us that *every* student who studies hard will learn to tango!

So let's change the quantifier from $\exists$ to $\forall$.

- $\forall x\ ((\text{Student}(x) \wedge \text{StudiesHard}(x)) \wedge \text{LearnsTango}(x))$

But now there's a problem with this translation. (What is it?)

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Everything is a studying, tangoing student ...?

$$\forall x\ ((\text{Student}(x) \wedge \text{StudiesHard}(x)) \wedge \text{LearnsTango}(x))$$

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The above sentence is just too strong: it says that everything is a student who studies hard and learns tango.

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The above sentence is just too strong: it says that everything is a student who studies hard and learns tango.

But that is obviously false. And it isn't what the dance studio wanted to say.

Everything is a studying, tangoing student ...?

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The above sentence is just too strong: it says that everything is a student who studies hard and learns tango.

But that is obviously false. And it isn't what the dance studio wanted to say.

How can we weaken it?

$\forall x\ ((\text{Student}(x) \wedge \text{StudiesHard}(x)) \wedge \text{LearnsTango}(x))$

Let's switch the $\wedge$ inside to a $\rightarrow$.

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Now it reads, in English:

Every student who studies hard will learn to tango.

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Now it reads, in English:

Every student who studies hard will learn to tango.

And that **is** what we were looking for.

Step-by-Step & Repair

What we just performed is the first way of approaching these more challenging translations. I'll call this the "Step-By-Step and Repair" method.

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We translated the sentence using the step by step method. But the translation that we got didn't work.

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What we just performed is the first way of approaching these more challenging translations. I'll call this the "Step-By-Step and Repair" method.

We translated the sentence using the step by step method. But the translation that we got didn't work.

So we had to tinker with it. We changed the quantifier to get that right, and then we had to change the interior connective.

Another Method: Paraphrasing In Advance

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Any student who studies hard will learn to tango.

Once we have done that paraphrasing in advance, the step-by-step method will work without any need for further adjustment afterwards.

Gaining Putin's Trust

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Example 2. If someone discos with him, Putin will trust that person.

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Example 2. If someone discos with him, Putin will trust that person.

Let's translate that sentence using the "Step by Step and Repair" method.

Putin on the Ritz ...

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This appears to be a conditional:

- Someone discos with Putin $\rightarrow$ Putin trusts that person.

Translating the Antecedent

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- Someone discos with Putin → Putin trusts that person

Antecedent: Someone discos with Putin

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Translate this, using

- Discos(x,y) as a predicate to express that x discos with y

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- Discos(x,y) as a predicate to express that x discos with y
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Translate this, using

- Discos(x,y) as a predicate to express that x discos with y
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Antecedent: ∃x Discos(x,putin)

Translating the Consequent

- $\exists$x Discos(x,putin) $\rightarrow$ Putin trusts that person

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- $\exists$x Discos(x,putin) $\rightarrow$ Putin trusts that person

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Consequent: Putin trusts that person

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Now, who is it again that Putin trusts?

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Ah yes. That person who discos with him.

Translating the Consequent

- $\exists$x Discos(x,putin) $\rightarrow$ Putin trusts that person

Now the consequent:

Consequent: Putin trusts that person

Now, who is it again that Putin trusts?

Ah yes. That person who discos with him.

We used the variable x for that person, so let's try using it again.

- Trusts(putin,x)

Completed Translation: First Try

- Someone discos with Putin $\rightarrow$ Putin trusts that person

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But now we have a big problem in this sentence.

Completed Translation: First Try

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- $\exists$x Discos(x,putin) $\rightarrow$ Trusts(putin,x)

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Can you see what it is?

Ooops. Scope Troubles.

The scope of the $\exists$ quantifier only extends to the end of the first complete wff after it.

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Let's try putting parentheses in to make the scope go all the way to the end of the sentence.

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- ∃x (Discos(x,putin) → Trusts(putin,x))

But now we have another problem. What is it?

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∃ and →: Don't Go Together Well

- ∃x (Discos(x,putin) → Trusts(putin,x))

Using a conditional as the main connective inside of an existential makes for a very weak sentence.

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An existential sentence is true whenever we can find at least one thing that makes the wff that follows it true.

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- This sentence will be true if we can find just one thing that makes the antecedent false.

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  - (Just one thing that *doesn't* disco with Putin makes the sentence true!)

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- This sentence will be true if we can find just one thing that makes the antecedent false.
  - (Just one thing that *doesn't* disco with Putin makes the sentence true!)
- It would also be true if we could find just one thing that makes the consequent true.

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  - (Just one thing that *doesn't* disco with Putin makes the sentence true!)
- It would also be true if we could find just one thing that makes the consequent true.
  - (Just one thing that Putin does trust makes the sentence true!)

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Hmmmm ...

So what can we do?

- ∃x (Discos(x,putin) → Trusts(putin,x))

Mark Criley Translations Requiring Paraphrasing

→ and ∀: Go together well

- ∃x (Discos(x,putin) → Trusts(putin,x))

We know that universals go better with →, so maybe we could try just switching the ∃ to a ∀.

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That gives us

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We know that universals go better with →, so maybe we could try just switching the ∃ to a ∀.

That gives us

- ∀x (Discos(x,putin) → Trusts(putin,x))

And what does that mean when we read it back in English?

Show a dictator some love

- ∀x (Discos(x,putin) → Trusts(putin,x))

Show a dictator some love

- $\forall x$ (Discos(x,putin) $\rightarrow$ Trusts(putin,x))

Now it's just a total inclusion sentence, totally including

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Subject: People who disco with Putin

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Predicate: People Putin trusts

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Filling these in we get

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Now it's just a total inclusion sentence, totally including

Subject: People who disco with Putin

within

Predicate: People Putin trusts

Filling these in we get

- Anyone who discos with Putin, Putin will trust.

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Or, paraphrase first

Again, if we realized that there was something a little fishy going on, we might have tried to paraphrase in English before translating into FOL.

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Again, if we realized that there was something a little fishy going on, we might have tried to paraphrase in English before translating into FOL.

If someone discos with Putin, Putin will trust that person.

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Putin will trust anyone who discos with him.

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or
Putin will trust anyone who discos with him.

Those paraphrases preserve the meaning of the original sentence, and will allow us to do the "step by step" method in the usual way.

A Sign of Trouble

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If **someone** discos with Putin,

A Sign of Trouble

Here's a clue that a sentence might need some paraphrasing:

It appears to be a conditional, but there is a pronoun that reaches across the "$\rightarrow$"

> If **someone** discos with Putin, Putin will trust **that person**.

Watch out!

And in general, you will want to watch out for uses of our usual English quantifier words ("some," "any," "a," etc.,) when they are embedded in English conditionals. They behave in non-standard ways.

Look Out Behind

Suppose your Driver's Ed teacher tells you:

Look Out Behind

Suppose your Driver's Ed teacher tells you:

Example 3. If anyone is behind you, don't back up!

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Since "any" usually means "every" and uses the universal quantifier, we might be tempted to say:

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Example 3. $\forall$x BackOf(x,me) $\rightarrow$ I don't back up!

Example 3. If anyone is behind you, don't back up!

Since "any" usually means "every" and uses the universal quantifier, we might be tempted to say:

Example 3. $\forall$x BackOf(x,me) $\rightarrow$ I don't back up!

But reading the example this way will lead to driving disaster!

Donkey Time!

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The following example is due to a famous discussion by the philosopher Peter Geach.

Donkey Time!

The following example is due to a famous discussion by the philosopher Peter Geach.

Example 4. Any farmer who owns a donkey ~~beats~~ vaccinates it.[1]

[1] We have cleaned it up a little bit for the ASPCA. No animals were harmed during the construction of this example.

Any farmer who owns a donkey vaccinates it.

What's weird about this sentence? Well, let's try translating it using the Step-by-Step and Repair method.

Step by Step

- Any farmer who owns a donkey vaccinates it.

Step by Step Method:

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Step by Step Method:

1. **Type:** Total Inclusion

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2. **Form:** $\forall x$ (Subject(x) $\rightarrow$ Predicate(x))

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Step by Step Method:

1. **Type:** Total Inclusion
2. **Form:** $\forall x$ (Subject(x) $\rightarrow$ Predicate(x))
3. **Identify Subject and Predicate:**

Step by Step

- Any farmer who owns a donkey vaccinates it.

Step by Step Method:

1. **Type:** Total Inclusion
2. **Form:** $\forall x$ (Subject(x) $\rightarrow$ Predicate(x))
3. **Identify Subject and Predicate:**
   - SUBJECT: x is a farmer who owns a donkey

Step by Step

- Any farmer who owns a donkey vaccinates it.

Step by Step Method:

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Mark Criley Translations Requiring Paraphrasing

Step By More Steps

Next, we translate the subject and predicate

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Step By More Steps

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PREDICATE: Vaccinates(x,y)

Substitute Translations

Now, we will put those translated sentences back into the original skeleton.

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Why not?

Scope Trouble Again

- $\forall x ((\text{Farmer}(x) \wedge \exists y (\text{Donkey}(y) \wedge \text{Owns}(x,y))) \rightarrow \text{Vaccinates}(x,y))$

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The quantifier for y only extends to the end of the first complete wff after it.

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So how far does the quantifier extend?

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So the variable *y* is free when it occurs in "Vaccinates (x,y)"

$\forall x\ ((\text{Farmer}(x) \wedge \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y))) \rightarrow \text{Vaccinates}(x,y))$

How can we fix this scope problem?

$\forall x\ ((\text{Farmer}(x) \wedge \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y))) \rightarrow \text{Vaccinates}(x,y))$

How can we fix this scope problem?

- How about moving all of the stuff about the donkey to the end of the sentence, so that it all comes together in the predicate?

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How can we fix this scope problem?

- How about moving all of the stuff about the donkey to the end of the sentence, so that it all comes together in the predicate?
- $\forall x\ (\text{Farmer}(x) \rightarrow \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y) \wedge \text{Vaccinates}(x,y)))$

Not quite right

Every farmer who owns a donkey vaccinates it.

Compare:

- $\forall x\ (\text{Farmer}(x) \rightarrow \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y) \wedge \text{Vaccinates}(x,y))$

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Compare:

- $\forall x\ (\text{Farmer}(x) \rightarrow \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y) \wedge \text{Vaccinates}(x,y))$

How does this read back?

Not quite right

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Compare:

- $\forall x\ (\text{Farmer}(x) \rightarrow \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y) \wedge \text{Vaccinates}(x,y))$

How does this read back?

Every farmer owns a donkey whom he vaccinates.

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- $\forall x\ (\text{Farmer}(x) \rightarrow \exists y\ (\text{Donkey}(y) \wedge \text{Owns}(x,y) \wedge \text{Vaccinates}(x,y))$

How does this read back?

Every farmer owns a donkey whom he vaccinates.

But we didn't **want** to say that every farmer owns a donkey. That is not part of the original sentence.

Back to the Drawing Board

The question was:

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How can we fix the scope problem in this sentence?

- $\forall x$ ((Farmer(x) $\wedge$ $\exists y$ (Donkey(y) $\wedge$ Owns(x,y))) $\rightarrow$ Vaccinates(x,y))

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∃ and → ...

But what's wrong with this?

∀x ∃y ((Farmer(x) ∧ (Donkey(y) ∧ Owns(x,y))) → Vaccinates(x,y))

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- ∃ and → don't go well together.

∃ and → ...

But what's wrong with this?

∀x ∃y ((Farmer(x) ∧ (Donkey(y) ∧ Owns(x,y))) → Vaccinates(x,y))

- ∃ and → don't go well together.
- They're very weak:
  - Find one thing that isn't a donkey that the farmer owns? The sentence turns out to be true.

How can we fix it? $\exists$ and $\wedge$ ...

We do know that the $\exists$ and the $\wedge$ go together nicely. So maybe we can change the $\rightarrow$ to a $\wedge$

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But if we read that sentence back, we can tell that it is too strong.

What does it say?

$\forall x \exists y ((Farmer(x) \wedge (Donkey(y) \wedge Owns(x,y))) \triangle Vaccinates(x,y))$

But if we read that sentence back, we can tell that it is too strong.

What does it say?

- Everything is a farmer and each farmer owns a donkey and vaccinates it.

Everything is a farmer?

No, not everything is a farmer.

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Our translation is false, even when our English sentence is true.

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Once again,

Everything is a farmer?

No, not everything is a farmer.

Our translation is false, even when our English sentence is true.

Once again,
we have failed.

One last shot ...

How about this? We didn't like this translation

$\forall x\ \exists y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

because $\exists$ and $\rightarrow$ don't go together well.

One last shot ...

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What else could we have done?

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So we changed the $\rightarrow$ to a $\wedge$

What else could we have done?

Change the $\exists$ to a $\forall$!

That is so crazy it just might work

$\forall x\ \exists y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

That is so crazy it just might work

$\forall x\ \exists y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

- now becomes

$\forall x\ \forall y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

It Works! It Works!

$\forall x\ \forall y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

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Read it back!

- Make any two choices of objects you like ...

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Read it back!

- Make any two choices of objects you like ...
- ...if the first choice is a Farmer ...

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Read it back!

- Make any two choices of objects you like ...
- ...if the first choice is a Farmer ...
- ...and the second one is a donkey

It Works! It Works!

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Read it back!

- Make any two choices of objects you like ...
- ...if the first choice is a Farmer ...
- ...and the second one is a donkey
- ...and that farmer owns that donkey

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- ...and that farmer owns that donkey
- ...then the farmer vaccinates that donkey

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- ...and that farmer owns that donkey
- ...then the farmer vaccinates that donkey

- Any farmer who owns a donkey vaccinates it.

Mark Criley Translations Requiring Paraphrasing

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Mark Criley Translations Requiring Paraphrasing

$\forall x\ \forall y\ ((\text{Farmer}(x) \wedge \text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vaccinates}(x,y))$

Also equivalent:

$\forall x\ (\text{Farmer}(x) \rightarrow \forall y\ ((\text{Donkey}(y) \wedge \text{Owns}(x,y)) \rightarrow \text{Vacc}(x,y)))$

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And even:

$\forall y\ (\text{Donkey}(y) \rightarrow \forall x\ ((\text{Farmer}(x) \wedge \text{Owns}(x,y)) \rightarrow \text{Vacc}(x,y)))$